Sublinear Time, Measurement-Optimal, Sparse Recovery For All

Martin J. Strauss

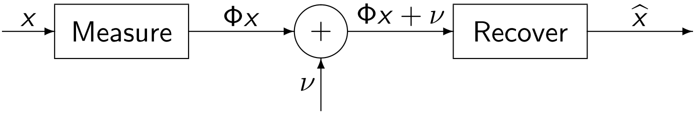
University of Michigan

Joint with Ely Porat, Bar Ilan

Outline

Sparse recovery

- $\widehat{x} = R(\Phi x + \nu) \approx x$
- Approximate best k-term signal; length is N



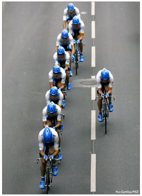
Some criteria of algorithms

Speed

Some criteria of algorithms

Speed

Accuracy

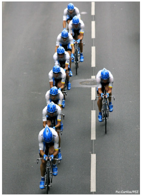


Some criteria of algorithms

Number of measurements

Speed

Accuracy



Some criteria of algorithms

- Number of measurements: want $O(k \log N/k) \approx \log \binom{N}{k}$

Some criteria of algorithms

- Number of measurements: want $O(k \log N/k) \approx \log \binom{N}{k}$
- Recovery runtime (speed):

Some criteria of algorithms

- Number of measurements: want $O(k \log N/k) \approx \log \binom{N}{k}$
- Recovery runtime (speed):
  - want poly($k \log N$).

Some criteria of algorithms

- Number of measurements: want $O(k \log N/k) \approx \log \binom{N}{k}$
- Recovery runtime (speed):
  - want poly$(k \log N)$.
  - Faster than previous *measurement-optimal* algorithms.

Some criteria of algorithms

- Number of measurements: want $O(k \log N/k) \approx \log \binom{N}{k}$
- Recovery runtime (speed):
  - want poly$(k \log N)$.
  - Faster than previous *measurement-optimal* algorithms.
  - ("sublinear time" algos lose to "superlinear" FFT every time.)

Some criteria of algorithms

- Number of measurements: want $O(k \log N/k) \approx \log \binom{N}{k}$
- Recovery runtime (speed):
  - want poly($k \log N$).
  - Faster than previous *measurement-optimal* algorithms.
  - ("sublinear time" algos lose to "superlinear" FFT every time.)
- Accuracy—how much error, which norm, universality...our model:

Some criteria of algorithms

- Number of measurements: want $O(k \log N/k) \approx \log \binom{N}{k}$
- Recovery runtime (speed):
  - want poly($k \log N$).
  - Faster than previous *measurement-optimal* algorithms.
  - ("sublinear time" algos lose to "superlinear" FFT every time.)
- Accuracy—how much error, which norm, universality...our model:
  - Recover *all* signals in (smaller) ℓ_1 ball, by *one* matrix.

Some criteria of algorithms

- Number of measurements: want $O(k \log N/k) \approx \log \binom{N}{k}$
- Recovery runtime (speed):
  - want poly$(k \log N)$.
  - Faster than previous *measurement-optimal* algorithms.
  - ("sublinear time" algos lose to "superlinear" FFT every time.)
- Accuracy—how much error, which norm, universality...our model:
  - Recover *all* signals in (smaller) ℓ_1 ball, by *one* matrix.
- Norm of error

Some criteria of algorithms

- Number of measurements: want $O(k \log N/k) \approx \log \binom{N}{k}$
- Recovery runtime (speed):
  - want poly($k \log N$).
  - Faster than previous *measurement-optimal* algorithms.
  - ("sublinear time" algos lose to "superlinear" FFT every time.)
- Accuracy—how much error, which norm, universality...our model:
  - Recover *all* signals in (smaller) ℓ_1 ball, by *one* matrix.
- Norm of error
  - Want ℓ_2:

$$\|x - \widehat{x}\|_2 \le \frac{\epsilon}{\sqrt{k}} \|x - x_k\|_1.$$

Some criteria of algorithms

- Number of measurements: want $O(k \log N/k) \approx \log \binom{N}{k}$
- Recovery runtime (speed):
  - want poly($k \log N$).
  - Faster than previous *measurement-optimal* algorithms.
  - ("sublinear time" algos lose to "superlinear" FFT every time.)
- Accuracy—how much error, which norm, universality...our model:
  - Recover *all* signals in (smaller) ℓ_1 ball, by *one* matrix.
- Norm of error
  - Want ℓ_2:
$$\|x - \widehat{x}\|_2 \leq \frac{\epsilon}{\sqrt{k}} \|x - x_k\|_1.$$
  - Here get only ℓ_1 (strictly worse):
$$\|x - \widehat{x}\|_1 \leq (1 + \epsilon)\|x - x_k\|_1.$$

Some results

Paper	No. meas.	time	norm
[GSTV07]	k polylog	poly$(k \log N)$	2
[Donoho04] [CRT04]	$k \log(N/k)$	poly(N)	2

Red is optimal.

Some results

Paper	No. meas.	time	norm
[GSTV07]	k polylog	poly$(k \log N)$	2
[Donoho04] [CRT04]	$k \log(N/k)$	poly(N)	2
[RI08]	$k \log(N/k)$	$N \log(N/k)$	1

Red is optimal.

Some results

Paper	No. meas.	time	norm
[GSTV07]	k polylog	poly$(k \log N)$	2
[Donoho04] [CRT04]	$k \log(N/k)$	poly(N)	2
[RI08]	$k \log(N/k)$	$N \log(N/k)$	1
Here/In progress	$k \log(N/k)$	poly$(k \log N)$	1

Red is optimal.

Here is `http://arxiv.org/abs/1012.1886v2`

Some results

Paper	No. meas.	time	norm
[GSTV07]	k polylog	$\mathrm{poly}(k \log N)$	2
[Donoho04] [CRT04]	$k \log(N/k)$	$\mathrm{poly}(N)$	2
[RI08]	$k \log(N/k)$	$N \log(N/k)$	1
Here/In progress	$k \log(N/k)$	$\mathrm{poly}(k \log N)$	1

Red is optimal.

Here is `http://arxiv.org/abs/1012.1886v2`

Other models by: Xu-Hassibi, Caldebank-Howard-Jafarpour, Gilbert-Li-P-S, ...

Group testing 1-sparse signals

Group testing on 1-sparse signal. First half or second? Recover bit-by-bit:

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

Group testing 1-sparse signals

Group testing on 1-sparse signal. First half or second? Recover bit-by-bit:

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \text{noise} \\ \text{noise} \\ 7 \\ \text{noise} \\ \text{noise} \\ \text{noise} \\ \text{noise} \\ \text{noise} \end{pmatrix}$$

Group testing 1-sparse signals

Group testing on 1-sparse signal. First half or second? Recover bit-by-bit:

$$\begin{pmatrix} 7 \\ 0 \\ 7 \\ 0 \end{pmatrix} \approx \begin{pmatrix} \text{Reference} \\ \text{Small} \\ \text{BIG} \\ \text{Small} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \text{noise} \\ \text{noise} \\ 7 \\ \text{noise} \\ \text{noise} \\ \text{noise} \\ \text{noise} \\ \text{noise} \end{pmatrix}$$

Techniques for some sublinear algorithms

- Hash into k buckets (hope to isolate HH's with low noise)

$$H = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- Group testing on 1-sparse signal.

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

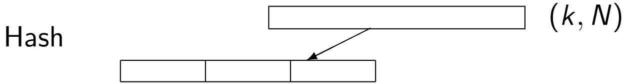
- Typically lose log factor in meas. Top row of H becomes:

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Outline

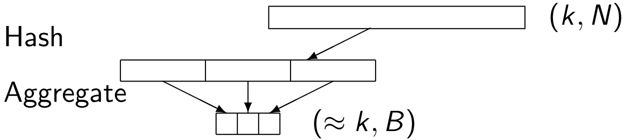
Algorithm

- Hash into $B = \sqrt{kN}$ buckets;



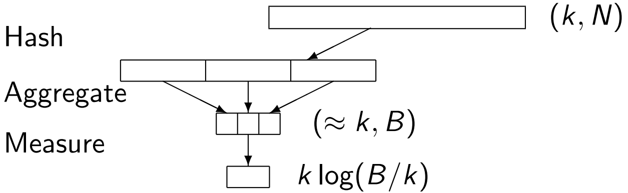
Algorithm

- Hash into $B = \sqrt{kN}$ buckets; Aggregate;



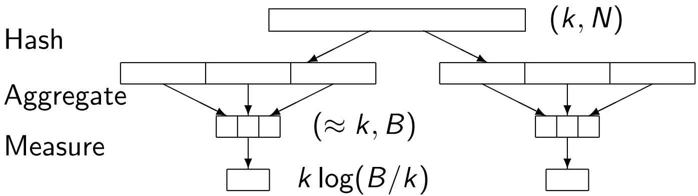
Algorithm

- Hash into $B = \sqrt{kN}$ buckets; Aggregate; Measure



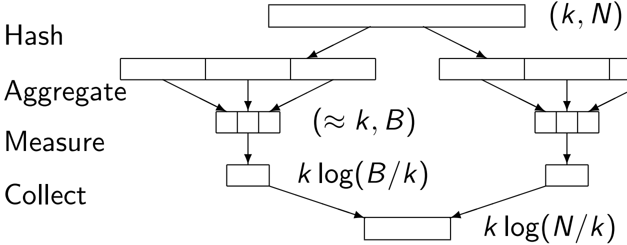
Algorithm

- Hash into $B = \sqrt{kN}$ buckets; Aggregate; Measure
- Repeat $\log(N/k)/\log(B/k) = 2$ times;



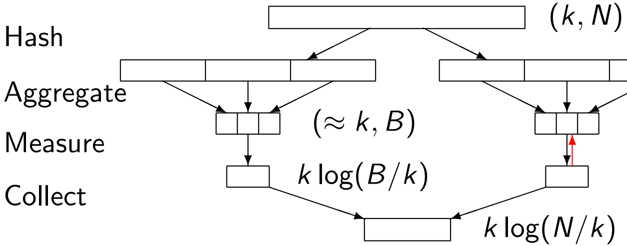
Algorithm

- Hash into $B = \sqrt{kN}$ buckets; Aggregate; Measure
- Repeat $\log(N/k)/\log(B/k) = 2$ times; collect measurements



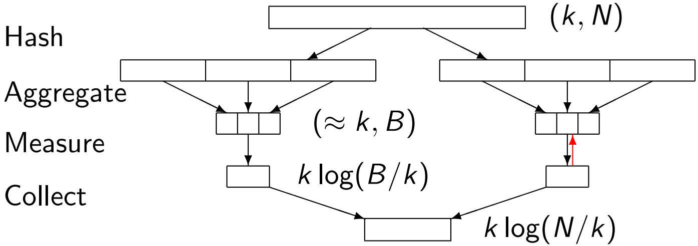
Algorithm

- Hash into $B = \sqrt{kN}$ buckets; Aggregate; Measure
- Repeat $\log(N/k)/\log(B/k) = 2$ times; collect measurements
- Recursively solve, naively.



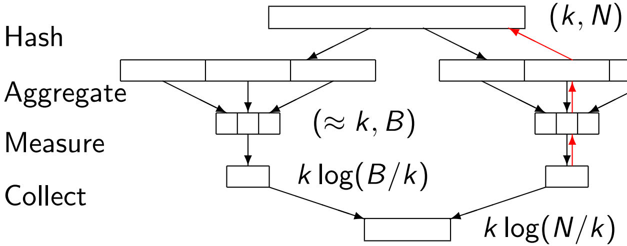
Algorithm

- Hash into $B = \sqrt{kN}$ buckets; Aggregate; Measure
- Repeat $\log(N/k)/\log(B/k) = 2$ times; collect measurements
- Recursively solve, naively. Time $\approx$ length $= B = \sqrt{kN}$



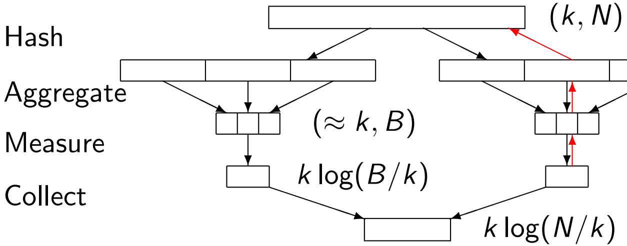
Algorithm

- Hash into $B = \sqrt{kN}$ buckets; Aggregate; Measure
- Repeat $\log(N/k)/\log(B/k) = 2$ times; collect measurements
- Recursively solve, naively. Time $\approx$ length $= B = \sqrt{kN}$
- Lift solution (from table).



Algorithm

- Hash into $B = \sqrt{kN}$ buckets; Aggregate; Measure
- Repeat $\log(N/k)/\log(B/k) = 2$ times; collect measurements
- Recursively solve, naively. Time $\approx$ length $= B = \sqrt{kN}$
- Lift solution (from table). Time $\approx$ no. preimages $= k(N/B) = \sqrt{kN}$



Outline

Result

Theorem

Algorithm takes $\approx \sqrt{kN}$ *time and uses* $k \log(N/k)$ *measurements.*

Result

Theorem

Algorithm takes $\approx \sqrt{kN}$ *time and uses* $k \log(N/k)$ *measurements.*

Need to show:

- Number of measurements and runtime—done.
- Correctness of Hashing procedure
  - Why $2 = \log(N/k)/\log(B/k)$ repetitions?
  - Why do we get $(\approx k, B)$-signal?
- Correctness of recursive solution—easy
- Correctness of lifting—easy by (lazy) design (use of table)

Correctness of Hashing

Lemma

Intermediate signal is indeed $\approx$ *k-sparse and length* B.

Correctness of Hashing

Lemma

Intermediate signal is indeed $\approx k$-sparse and length B.

- Each heavy hitter is isolated except with prob k/B.

Correctness of Hashing

Lemma

Intermediate signal is indeed $\approx$ k-sparse and length B.

- Each heavy hitter is isolated except with prob k/B.
  - $\geq k/2$ fail with prob $(k/B)^k = 2^{-k\log(B/k)}$

Correctness of Hashing

Lemma

Intermediate signal is indeed $\approx k$-sparse and length B.

- Each heavy hitter is isolated except with prob k/B.
  - $\geq k/2$ fail with prob $(k/B)^k = 2^{-k\log(B/k)}$
- Heavy hitters land in set S of about k of B buckets. Consider t noise items of size $1/t$:

Correctness of Hashing

Lemma

Intermediate signal is indeed $\approx k$-sparse and length B.

- Each heavy hitter is isolated except with prob k/B.
  - $\geq k/2$ fail with prob $(k/B)^k = 2^{-k\log(B/k)}$
- Heavy hitters land in set S of about k of B buckets. Consider t noise items of size $1/t$:
  - Each noise item lands in S with prob k/B

Correctness of Hashing

Lemma

Intermediate signal is indeed $\approx$ k-sparse and length B.

- Each heavy hitter is isolated except with prob k/B.
  - $\geq k/2$ fail with prob $(k/B)^k = 2^{-k\log(B/k)}$
- Heavy hitters land in set S of about k of B buckets. Consider t noise items of size $1/t$:
  - Each noise item lands in S with prob k/B
  - $\geq t/2$ noise items land in S with prob $(k/B)^t = 2^{-t\log(B/k)}$ (Otherwise, enough of S survives)

Correctness of Hashing

Lemma

Intermediate signal is indeed $\approx k$-sparse and length B.

- Each heavy hitter is isolated except with prob k/B.
  - $\geq k/2$ fail with prob $(k/B)^k = 2^{-k\log(B/k)}$
- Heavy hitters land in set S of about k of B buckets. Consider t noise items of size $1/t$:
  - Each noise item lands in S with prob k/B
  - $\geq t/2$ noise items land in S with prob $(k/B)^t = 2^{-t\log(B/k)}$ (Otherwise, enough of S survives)
- Repeat $\log(N/k)/\log(B/k)$ times

Correctness of Hashing

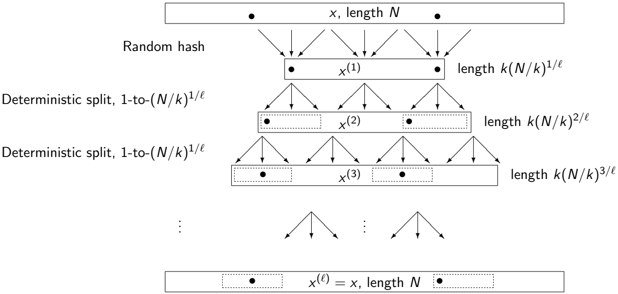
Lemma

Intermediate signal is indeed $\approx$ k-sparse and length B.

- Each heavy hitter is isolated except with prob k/B.
  - $\geq k/2$ fail with prob $(k/B)^k = 2^{-k\log(B/k)}$
- Heavy hitters land in set S of about k of B buckets. Consider t noise items of size $1/t$:
  - Each noise item lands in S with prob k/B
  - $\geq t/2$ noise items land in S with prob $(k/B)^t = 2^{-t\log(B/k)}$ (Otherwise, enough of S survives)
- Repeat $\log(N/k)/\log(B/k)$ times
  - Failure probs drop to $(k/N)^k \leq \binom{N}{k}^{-1}$ and $(k/N)^t \leq \binom{N}{t}^{-1}$
  - Take union bound.

More generally...

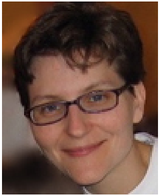
- Cascade through any chosen number ℓ of levels.
- poly(ℓ) problems with parameters $(k, k(N/k)^{1/\ell})$
- Time around poly(ℓ)$k(N/k)^{1/\ell}$
- Number of measurements is around poly(ℓ)$k \log(N/k)$



Outline

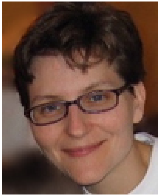
What's next?

(Joint with Anna Gilbert, Yi Li, Ely Porat)

What's next?

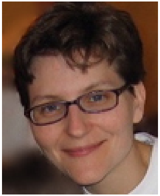
(Joint with Anna Gilbert, Yi Li, Ely Porat)



- Avoid lookup table

What's next?

(Joint with Anna Gilbert, Yi Li, Ely Porat)



- Avoid lookup table
- Lower runtime from $\mathrm{poly}(\ell)k(N/k)^{1/\ell}$ to $\mathrm{poly}(k, \log N)$

Avoid lookup table

- When we recover heavy hitter i,

Avoid lookup table

- When we recover heavy hitter i,
  - ...can also get arbitrary $O(\log(B/k))$-bit message!

Avoid lookup table

- When we recover heavy hitter i,
  - ...can also get arbitrary $O(\log(B/k))$-bit message!
- (Partially) codes pointer back to $i' \in [N]$.

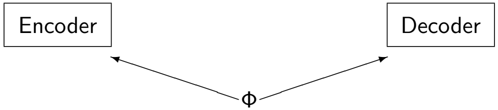
Avoid lookup table

- When we recover heavy hitter i,
  - ...can also get arbitrary $O(\log(B/k))$-bit message!
- (Partially) codes pointer back to $i' \in [N]$.
  - No need to store back pointer: $[B] \to [N]$ explicitly in table.

Avoid lookup table

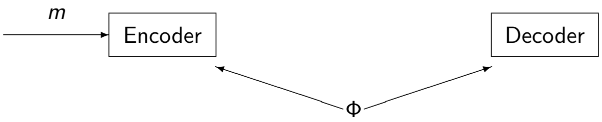
- When we recover heavy hitter i,
  - ...can also get arbitrary $O(\log(B/k))$-bit message!
- (Partially) codes pointer back to $i' \in [N]$.
  - No need to store back pointer: $[B] \to [N]$ explicitly in table.
  - Only need to use hash function *forwards*: $[N] \to [B]$.

Sparse recovery channel—The medium is the message



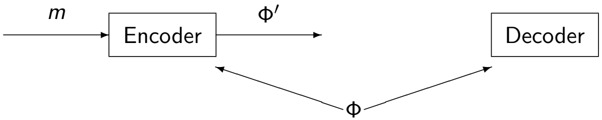
- Encoder and Decoder agree on Φ (independent of message)

Sparse recovery channel—The medium is the message



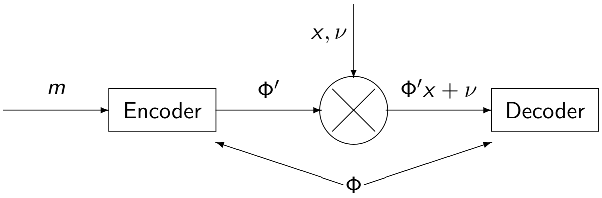
- Encoder and Decoder agree on Φ (independent of message)
- Message m of length B and alphabet size B/k

Sparse recovery channel—The medium is the message



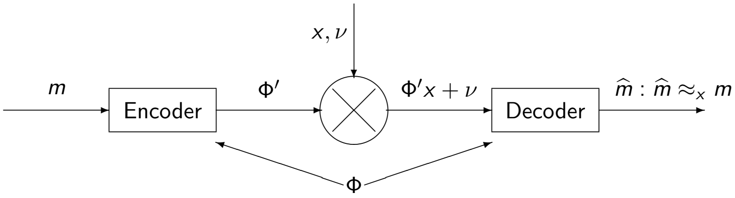
- Encoder and Decoder agree on Φ (independent of message)
- Message m of length B and alphabet size B/k
- Encoder makes final measurement matrix Φ' from Φ and m

Sparse recovery channel—The medium is the message



- Encoder and Decoder agree on Φ (independent of message)
- Message m of length B and alphabet size B/k
- Encoder makes final measurement matrix Φ' from Φ and m
- Channel picks x and $\nu(\neq 0?)$ and produces $\Phi' x + \nu$.

Sparse recovery channel—The medium is the message



- Encoder and Decoder agree on Φ (independent of message)
- Message m of length B and alphabet size B/k
- Encoder makes final measurement matrix Φ' from Φ and m
- Channel picks x and $\nu (\neq 0?)$ and produces $\Phi' x + \nu$.
- Decoder tries to recover m in x-weighted sense; need $\widehat{m}_i \approx m_i$ for many i such that $|x_i|$ is large. (Decoder doesn't know x.)

Coding one bit

(Reduced group testing.) Hash into k buckets. One bucket:

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

1-bit message

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Leads to

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

Coding one bit

(Reduced group testing.) Hash into k buckets. One bucket:

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

1-bit message

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Leads to

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

- With $k \log(B/k)$ measurements, $\log(B/k)$ lossy chances to code bits.

Coding one bit

(Reduced group testing.) Hash into k buckets. One bucket:

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

1-bit message

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Leads to

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

- With $k \log(B/k)$ measurements, $\log(B/k)$ lossy chances to code bits.
- With ECC, get $\log(B/k)(\approx \log N?)$-bit msg for each HH.

Coding one bit

(Reduced group testing.) Hash into k buckets. One bucket:

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

1-bit message

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Leads to

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

- With $k \log(B/k)$ measurements, $\log(B/k)$ lossy chances to code bits.
- With ECC, get $\log(B/k)(\approx \log N?)$-bit msg for each HH.

Use message to lift solution, rather than explicit lookup table.

Three kinds of information

Algorithm:

- Hash into B buckets
- Repeat $r = \log(N/k)/\log(N/B)$ times
- Solve recursively

Need $\log N$ bits of backpointer $\text{hash}^{-1} :\to [N]$.

Three kinds of information

Algorithm:

- Hash into B buckets
- Repeat $r = \log(N/k)/\log(N/B)$ times
- Solve recursively

Need $\log N$ bits of backpointer $\text{hash}^{-1} : \to [N]$.
j'th repetition, $j = 1, 2, \ldots, r$, gives tuple of

- $\log(B/k)$ codeable bits m_i
- j (side information)
- Index $i_j \in [B]$ of recursive heavy hitter in j'th repetition ($\log B$ non-codeable bits)

Three kinds of information

Algorithm:

- Hash into B buckets
- Repeat $r = \log(N/k)/\log(N/B)$ times
- Solve recursively

Need $\log N$ bits of backpointer $\text{hash}^{-1} :\to [N]$.
j'th repetition, $j = 1, 2, \ldots, r$, gives tuple of

- $\log(B/k)$ codeable bits m_i
- j (side information)
- Index $i_j \in [B]$ of recursive heavy hitter in j'th repetition ($\log B$ non-codeable bits)

Code payload and linking information into m_i and assemble.

Three kinds of information

Algorithm:

- Hash into B buckets
- Repeat $r = \log(N/k)/\log(N/B)$ times
- Solve recursively

Need $\log N$ bits of backpointer $\text{hash}^{-1} :\rightarrow [N]$.
j'th repetition, $j = 1, 2, \ldots, r$, gives tuple of

- $\log(B/k)$ codeable bits m_i
- j (side information)
- Index $i_j \in [B]$ of recursive heavy hitter in j'th repetition ($\log B$ non-codeable bits)

Code payload and linking information into m_i and assemble.
How?

Outline

Network coding

First, network (rateless) coding:

Network coding

First, network (rateless) coding:

- Message (movie) of length n

Network coding

First, network (rateless) coding:

- Message (movie) of length n
- Download for later viewing (not streamed, not DVD by mail, ...)

Network coding

First, network (rateless) coding:

- Message (movie) of length n
- Download for later viewing (not streamed, not DVD by mail, ...)
- Flaky network—dropped connections (erasures) but *no errors*

Network coding

First, network (rateless) coding:

- Message (movie) of length n
- Download for later viewing (not streamed, not DVD by mail, ...)
- Flaky network—dropped connections (erasures) but *no errors*
- Publisher breaks message into p packets, encodes, and broadcasts continually

Network coding

First, network (rateless) coding:

- Message (movie) of length n
- Download for later viewing (not streamed, not DVD by mail, ...)
- Flaky network—dropped connections (erasures) but *no errors*
- Publisher breaks message into p packets, encodes, and broadcasts continually
- Subscriber needs *any* $O(p)$ packets to recover message.

Network coding

First, network (rateless) coding:

- Message (movie) of length n
- Download for later viewing (not streamed, not DVD by mail, ...)
- Flaky network—dropped connections (erasures) but *no errors*
- Publisher breaks message into p packets, encodes, and broadcasts continually
- Subscriber needs *any* $O(p)$ packets to recover message.
- Punchline, e.g.,

Network coding

First, network (rateless) coding:

- Message (movie) of length n
- Download for later viewing (not streamed, not DVD by mail, ...)
- Flaky network—dropped connections (erasures) but *no errors*
- Publisher breaks message into p packets, encodes, and broadcasts continually
- Subscriber needs *any* $O(p)$ packets to recover message.
- Punchline, e.g.,
    - Send ever-new points on graph of degree-p polynomial.

Network coding

First, network (rateless) coding:

- Message (movie) of length n
- Download for later viewing (not streamed, not DVD by mail, ...)
- Flaky network—dropped connections (erasures) but *no errors*
- Publisher breaks message into p packets, encodes, and broadcasts continually
- Subscriber needs *any* $O(p)$ packets to recover message.
- Punchline, e.g.,
  - Send ever-new points on graph of degree-p polynomial.
  - Any $p + 1$ points suffice.

Multiple-stream network coding problem

- Unordered set of k messages (movies), length n, transmitted simultaneously.

Multiple-stream network coding problem

- Unordered set of k messages (movies), length n, transmitted simultaneously.
  Total kn bits. Want to recover from $O(nk)$ total bits, avoiding $\log k$ header bits (which movie?) per packet.

Multiple-stream network coding problem

- Unordered set of k messages (movies), length n, transmitted simultaneously.
  Total kn bits. Want to recover from $O(nk)$ total bits, avoiding $\log k$ header bits (which movie?) per packet.

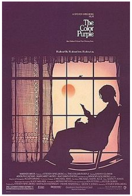
Get error correction for free! Why?

Multiple-stream network coding problem

- Unordered set of k messages (movies), length n, transmitted simultaneously.
  Total kn bits. Want to recover from $O(nk)$ total bits, avoiding $\log k$ header bits (which movie?) per packet.

Get error correction for free! Why?

Packets from one movie

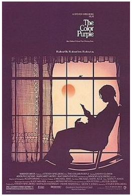


Multiple-stream network coding problem

- Unordered set of k messages (movies), length n, transmitted simultaneously.
  Total kn bits. Want to recover from $O(nk)$ total bits, avoiding $\log k$ header bits (which movie?) per packet.

Get error correction for free! Why?

Packets from one movie



can be regarded as noise in another

Upcoming results

Theorem

There's an algorithm that runs in time $k \log^{O(1)} N$, uses $O(k \log N/k)$ measurements, and returns $\widehat{x}$ with

$$\|x - \widehat{x}\|_1 \leq (1 + \epsilon)\|x - x_k\|_1.$$

(Joint with Anna Gilbert, Yi Li, Ely Porat)

Upcoming results

Theorem

There's an algorithm that runs in time $k \log^{O(1)} N$, uses $O(k \log N/k)$ measurements, and returns $\widehat{x}$ with

$$\|x - \widehat{x}\|_1 \le (1 + \epsilon)\|x - x_k\|_1.$$

(Joint with Anna Gilbert, Yi Li, Ely Porat)
Can this be improved with better error-correcting codes?

Outline

1. Preliminaries
   - Problem we are addressing
2. Algorithm
3. Result and Analysis
4. Next Results
   - Avoid lookup table
   - Faster runtime
5. Network Coding—wake up!
   - Conclusion
6. Conclusion

Conclusion

- First sublinear-time algo with optimal measurements in forall model, with
$$\|x - \widehat{x}\|_1 \leq (1 + \epsilon)\|x - x_k\|_1$$

Conclusion

- First sublinear-time algo with optimal measurements in forall model, with
$$\|x - \widehat{x}\|_1 \leq (1 + \epsilon)\|x - x_k\|_1$$

- Time $\sqrt{kN}$, improveable (?) to $\mathrm{poly}(k, \log N)$

Conclusion

- First sublinear-time algo with optimal measurements in forall model, with
$$\|x - \widehat{x}\|_1 \leq (1+\epsilon)\|x - x_k\|_1$$
- Time $\sqrt{kN}$, improveable (?) to $\mathrm{poly}(k, \log N)$
- Lookup table of size $Nk^{1/4}$, removeable (?)

Conclusion

- First sublinear-time algo with optimal measurements in forall model, with
$$\|x - \widehat{x}\|_1 \leq (1+\epsilon)\|x - x_k\|_1$$
- Time $\sqrt{kN}$, improveable (?) to $\text{poly}(k, \log N)$
- Lookup table of size $Nk^{1/4}$, removeable (?)

Finale is open: Improve to 2-norm:

$$\|x - \widehat{x}\|_2 \leq \frac{\epsilon}{\sqrt{k}}\|x - x_k\|_1.$$